$$\textbf{\textit{Packing fraction}} = \frac{\textbf{\textit{Total particle volume}}}{\textbf{\textit{Container volume}}}$$

Packing fraction = 1

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FCC Packing Fraction Calculation

Face Centered Cubic (FCC):

Number of spheres per cube: (6 x ½) + (8 x 1/8) = 4

$$Sphere\ Volume = \frac{4}{3}\pi r^3 \times 4$$

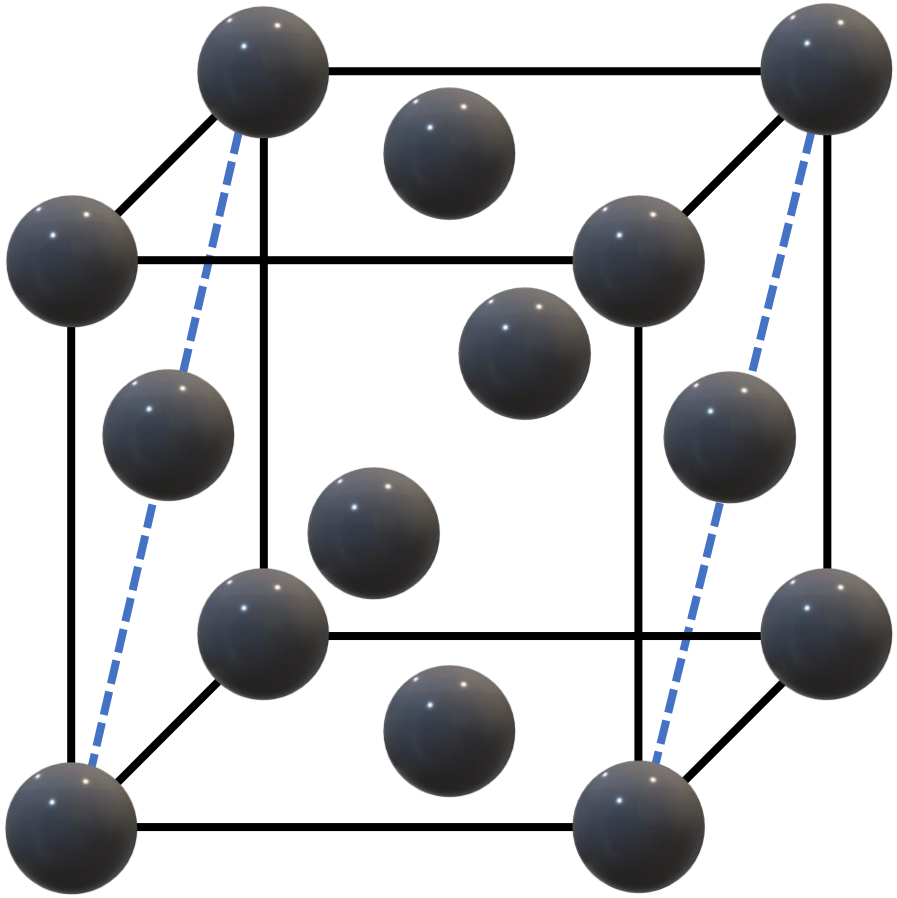
$$Cube\ Volume = a^3$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = \text{Packing fraction}$$

For close packing, 4 sphere radii (r) per face diagonal (blue dashed line).

$$4r = \sqrt{2a^2}$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = 0.740\ \ldots$$



Packing fraction = 0.74 and porosity = 0.26 (26%)

BCC Packing Fraction Calculation

Body Centered Cubic (FCC):

Number of spheres per cube: (1 x 1) + (8 x 1/8) = 2

$$Sphere\ Volume = \frac{4}{3}\pi r^3 \times 2$$

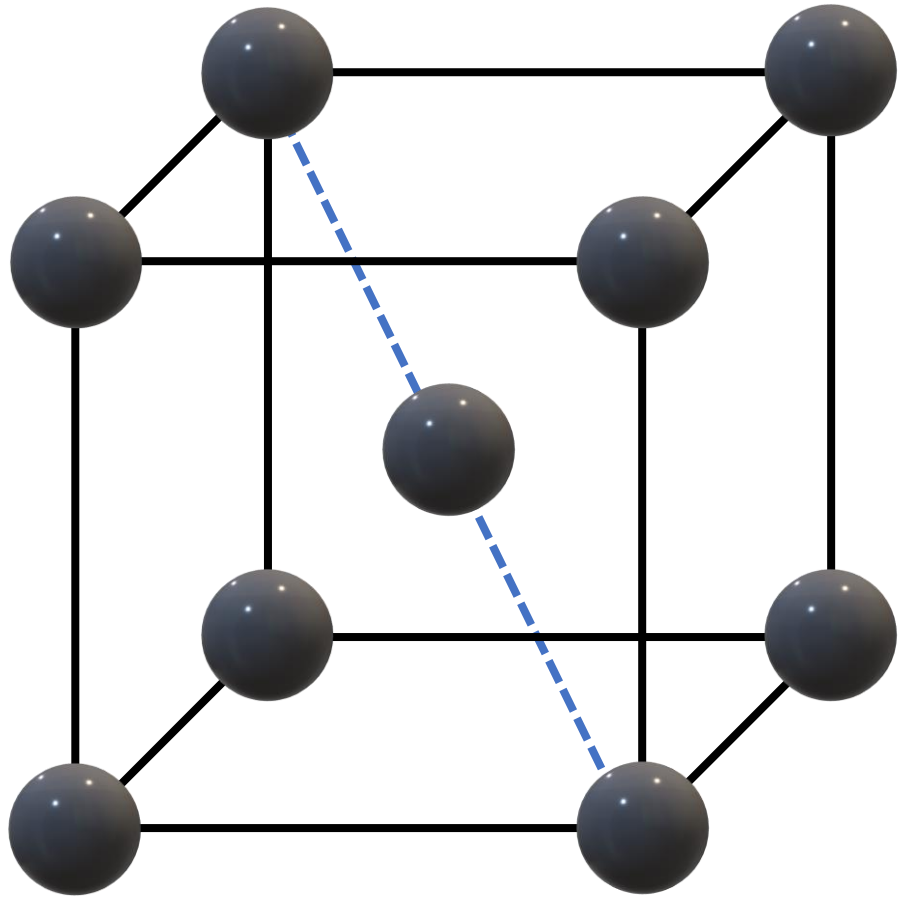
$$Cube\ Volume = a^3$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = \text{Packing fraction}$$

For close packing, 4 sphere radii (r) per cube diagonal (blue dashed line).

$$4r = \sqrt{3a^2}$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = 0.68\ \ldots$$



Packing fraction = 0.68 and porosity = 0.32 (32%)

Simple Cubic Packing Fraction Calculation

Number of spheres per cube: (8 x 1/8) = 1

$$Sphere\ Volume = \frac{4}{3}\pi r^3$$

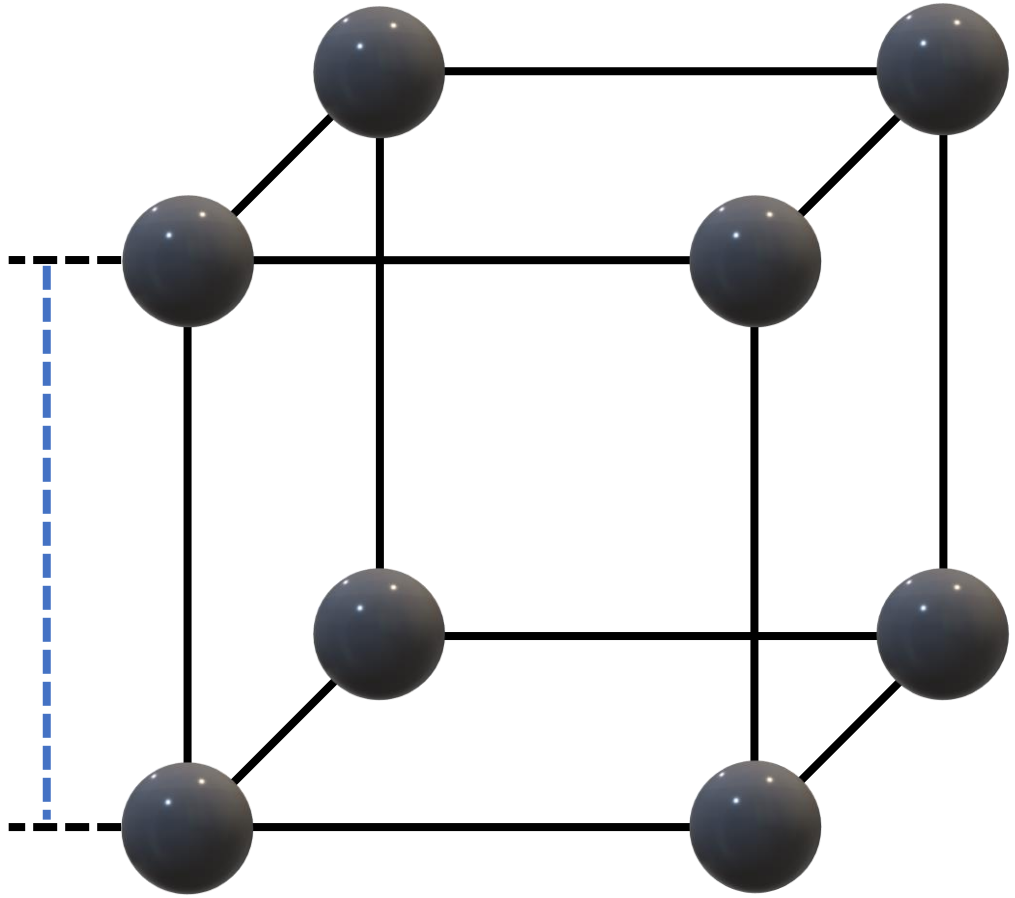
$$Cube\ Volume = a^3$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = \text{Packing fraction}$$

For close packing, 2 sphere radii (r) per cube length (blue dashed line).

$$2r = a$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = 0.52\ \ldots$$



Packing fraction = 0.52 and porosity = 0.48 (48%)

Simple Cubic Packing Fraction Calculation

Number of spheres per cube: (8 x 1/8) = 1

$$Sphere\ Volume = \frac{4}{3}\pi r^3$$

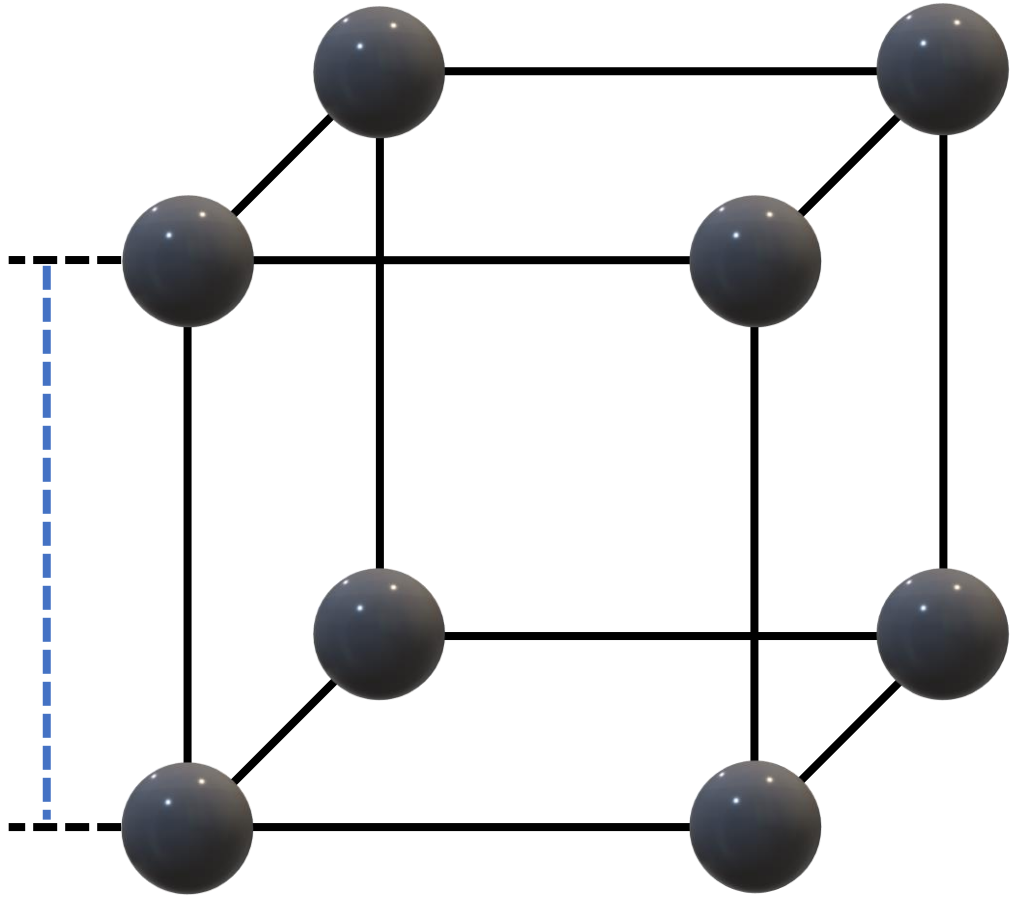
$$Cube\ Volume = a^3$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = \text{Packing fraction}$$

For close packing, 2 sphere radii (r) per cube length (blue dashed line).

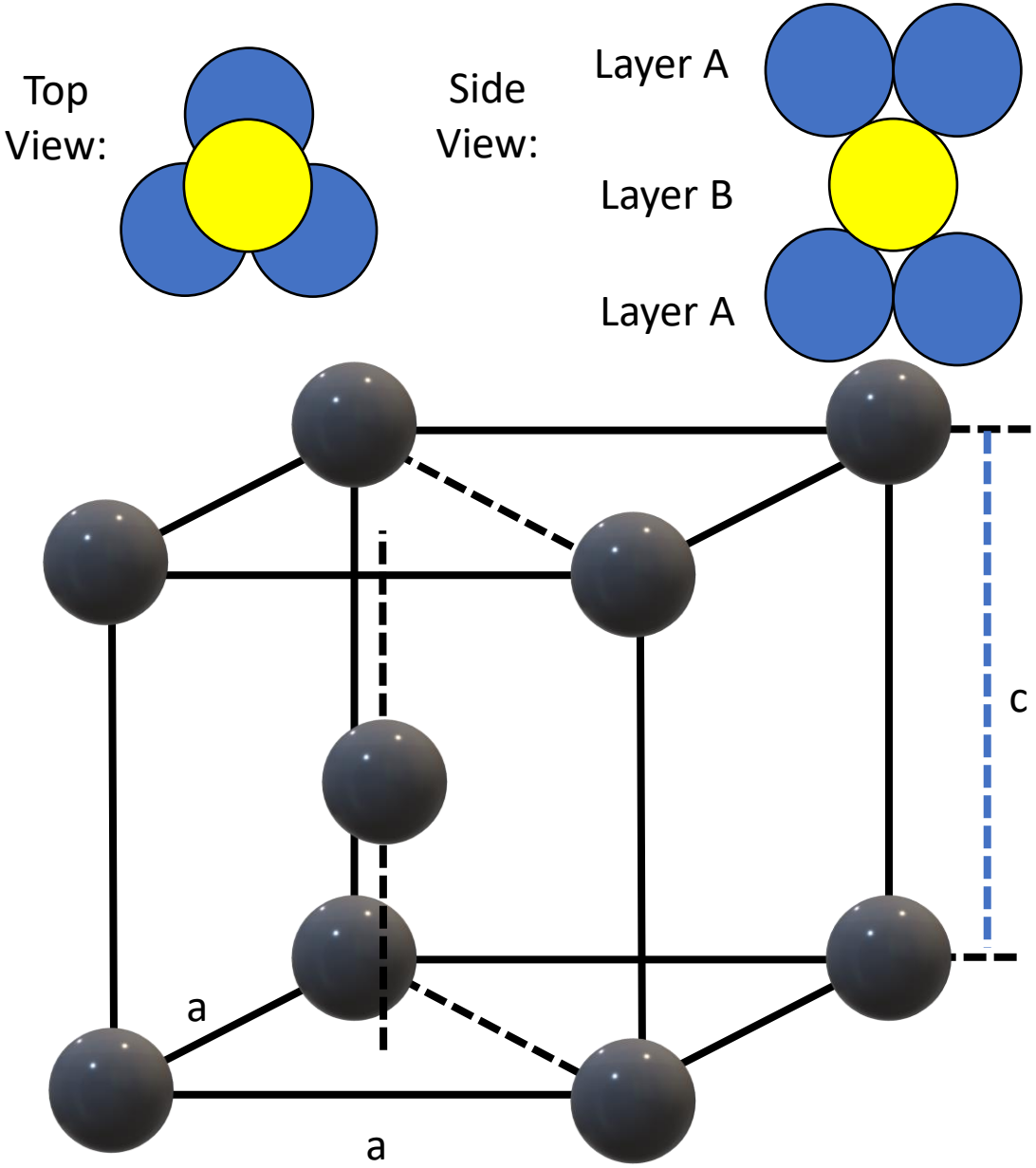
$$2r = a$$

$$\frac{Sphere\ Volume}{Cube\ Volume} = 0.52\ \ldots$$



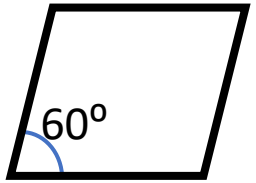
Packing fraction = 0.52 and porosity = 0.48 (48%)

HCP Packing Fraction Calculation



Number of spheres per volume: (1 x 1) + 1 = 2

$Sphere\ Volume = \frac{4}{3}\pi r^3$ x 2



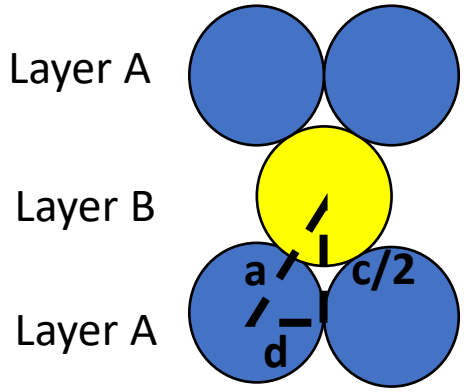
$Volume = a^2 \sin(60^0) \times c$

$\frac{Sphere\ Volume}{Volume}$ = Packing fraction

For close packing, it's a more complicated relationship between volume parameters and sphere parameters...

$$\frac{\frac{4}{3}\pi r^3 \text{ x } 2}{a^2 \sin(60^0) \times c} = Packing\ fraction$$

HCP Packing Fraction Calculation



1) Relating a, c and d:

$$a^2 = \left(\frac{c}{2}\right)^2 + d^2$$

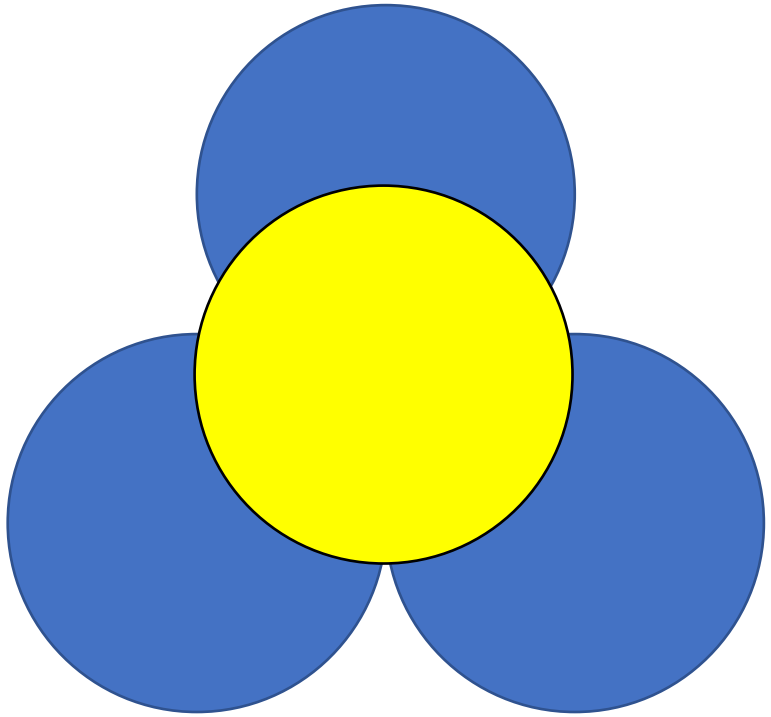
2) Relating a and d:

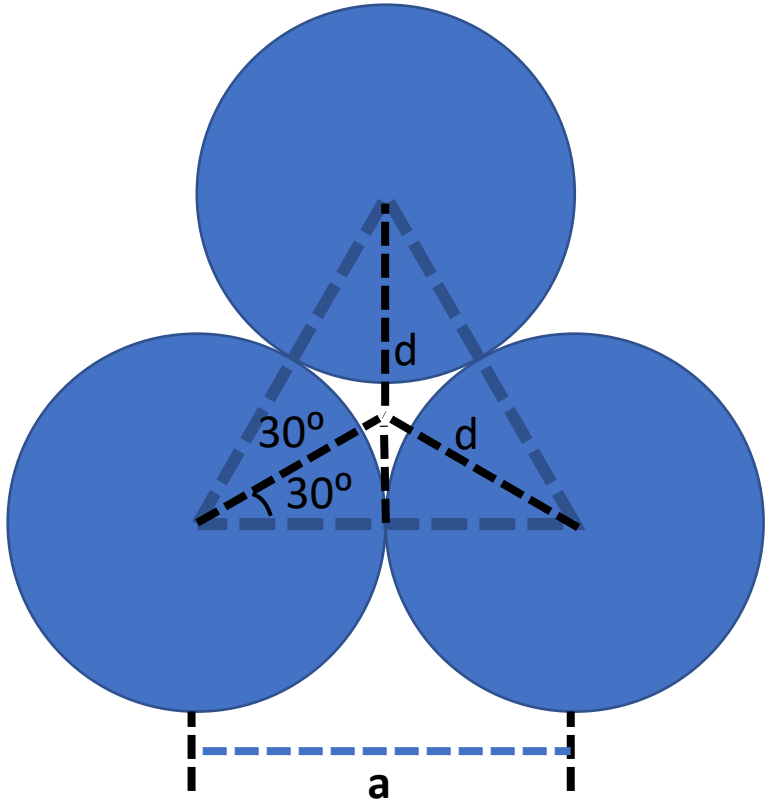
$$\frac{a}{2} = \mathrm{d} \times \cos(30^0)$$

$$\mathrm{a} = \mathrm{d}\sqrt{3}$$

3) Relating a and c:

Substitute 2 into 1 to obtain $\mathrm{a} = c\sqrt{\frac{8}{3}}$





HCP Packing Fraction Calculation

Equations:

$$a = c\sqrt{\frac{8}{3}}$$

$$a = 2r$$

$$Sphere\ Volume = \frac{4}{3}\pi r^3 \text{ x } 2$$

$$Volume = a^2 \sin(60^0) \times c$$

$$\frac{Sphere\ Volume}{Volume} = Packing\ fraction$$

$$\frac{\frac{4}{3}\pi r^3 \text{ x } 2}{a^2 \sin(60^0) \times c} = Packing\ fraction$$

Substituting in 1) and 2) into the packing fraction equation yields:

$$\frac{\pi}{3\sqrt{2}} = Packing\ fraction = 0.74\ (Same\ as\ FCC)$$

Structure	Packing Fraction
FCC	0.74
BCC	0.68
HCP	0.74
Simple Cubic	0.52

Structure	Porosity (%)
FCC	26
BCC	32
HCP	26
Simple Cubic	48